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# Interpretations of quantum mechanics and some claimed resolutions of the EPR paradox

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**Abstract.** We discuss a number of papers which claim that the EPR 'paradox' may be resolved using density-matrix methods. We first show that it is only inside the Copenhagen interpretation that the EPR experiment has any appearance of paradox; inside an ensemble interpretation, or hidden-variable theory, it may be analysed in a straightforward way. We then show that these 'resolutions' are implicitly using an ensemble interpretation.

## 1. Introduction

Recently we commented (Whitaker and Singh 1981) (to be referred to as I) on a paper by Cantrell and Scully (1978) (to be referred to as CS); these authors had claimed to resolve the well known EPR 'paradox' (Einstein *et al* 1935) by use of the reduced density matrix. We aimed to show that a density-matrix treatment could only give the same result as one using wavefunctions.

Nevertheless, it does appear that the idea that the density matrix plays a special role in quantum measurement, has a more widespread presence in the literature.

Törnqvist (1981), for example, states that it is well known that the EPR 'paradox' can be 'formally resolved within non-relativistic quantum mechanics using e.g. density-matrix language'. Ramachandran and Murthy (1980) state that 'the density matrix also plays a fundamental role in relation to the quantum theory of measurement'. Both papers refer to CS, the second also to a very general discussion of measurement given by Wigner (1971).

More significant is the claimed resolution of the EPR 'paradox' given by Jauch in his important book (Jauch 1968). Throughout his book, Jauch works almost entirely in terms of density matrices. CS follow his treatment fairly closely.

We reiterate here our belief, expressed in I, that use of the density matrix as such cannot affect any result of the theory. We think that any results that these treatments can justifiably claim are a result of the fact that they are implicitly using an ensemble interpretation of quantum mechanics, rather than a Copenhagen interpretation. It is well known that, under an ensemble interpretation, the EPR 'paradox' is not paradoxical at all.

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In the following section of this paper, we discuss the various interpretations, and how they handle the EPR experiment; in the third section we analyse Jauch's treatment.

## 2. Interpretations of quantum mechanics and the EPR experiment

Let us first clarify the differences between different interpretations of quantum mechanics. Essentially we follow Ballentine (1970) (to be referred to as B), though we adjust the terminology in one respect. Different classifications are discussed later. His definition of the Copenhagen interpretation is that it states that a wavefunction can provide a complete description of a single system. It therefore leads to the drastic phenomenon of wavefunction collapse at a measurement. Thus it experiences great difficulties in describing experiments of the EPR type, as there is little alternative to accepting that a measurement on one particle at one position causes wavefunction collapse, not only for that particle, but also for a second particle no longer interacting with the first.

EPR claim that this conflicts with our ideas of 'physical reality'. Since the wavefunction collapse is supposed to be simultaneous for the two particles, there is in any case an apparent conflict with the theory of relativity, as Einstein (1928) hinted remarkably early in the history of quantum mechanics, before the development of the EPR 'paradox' itself. Upholders of the Copenhagen interpretation may, of course, deny that difficulties exist; among others, Bohr (1935) has protested that the result is perfectly natural, and does not threaten the coherence of the Copenhagen interpretation.

We group together a variety of approaches as 'ensemble interpretations'. Let us start with that advocated strongly by Ballentine (B), which is along the lines sketched by Einstein (1949). It maintains that quantum mechanics cannot describe individual systems.

Under this title we include hidden-variable theories. Jammer (1974) points out that hidden variables are not a necessary component of an ensemble interpretation. He states that Einstein, for instance, never actually endorsed a hidden-variable theory, but hoped for a more complete departure from the orthodox approach. But it is obvious that hidden variables fit well into ensemble interpretations; they help to explain why it is necessary to consider an ensemble in the first place. One is performing the calculations over the range of values of the hidden variables. While interest in hidden-variable theories has never disappeared, their modern revival was mainly due to Bohm (1952a, b) (see also Bohm and Vigier 1954).

We have given our terminology for the various interpretations, but we recognise that different authors use different names for the same interpretation, and/or the same name for different interpretations. Ballentine, in fact, uses the term 'statistical interpretation' for our ensemble interpretation, while other authors appear to use the term 'Copenhagen interpretation' for what we would call, again, the ensemble interpretation. Belinfante (1973), for instance, writes 'the basic interpretation of quantum theory is what Ballentine (1970) calls the statistical interpretation. By Copenhagen interpretation we mean essentially the statistical interpretation together with the belief that it is not advantageous to introduce hidden variables as long as there is no experimental evidence necessitating their introduction'. d'Espagnat (1976) writes that 'most of the predictions of quantum mechanics are of a statistical nature and therefore make sense only for ensembles'.

When we turn to discussion of quantum mechanical measurement, we will realise that ensemble interpretations (including hidden-variable theories) help us to avoid the difficulties associated with wavefunction collapse. Let us consider the EPR experiment. We use, in fact, the Bohm (1951) modification, in which two spin-$\frac{1}{2}$ particles (with total spin angular momentum zero) separate, moving in opposite directions on, say, the $y$ axis. The $x$ or $z$ component of the spin of one of the particles is measured. If it is the $z$ component of the spin of particle 1 that is measured all that the ensemble interpretation can tell us is that, for the ensemble of systems, half the values found will be $+\frac{1}{2}$, and half $-\frac{1}{2}$. The inference drawn for particle 2 is the equally uninteresting one that, again, over the ensemble, half the values of $S_z$ will be $+\frac{1}{2}$ and half $-\frac{1}{2}$. There is obviously, as stressed by Ballentine, not a vestige of a paradox.

(We should however note that although, as we have already mentioned, d'Espagnat (1976) applies the laws of quantum mechanics only to ensembles (not to individual systems, except in the special case where certain consequences may be predicted for every member of an ensemble), he *is* able to discuss the EPR experiment for an individual system via the concept of the 'element of reality'.)

When one uses hidden-variable theories, one can be more specific about individual systems. Discussing the EPR experiment using the most naive hidden-variable theories, one might actually say that, for each system, the two particles have (equal and opposite) definite, though unknown, values of $S_z$ and $S_x$, which the experiment will ascertain. Thus the observer does not have to 'create' the physical situation he observes, a suggestion that Ballentine describes as 'solipsism'.

Of course, it is easy to challenge such simple conceptions of hidden variables (Jauch 1968 p 112), and, as is well known, Bell's theorem (Bell 1964, Wigner 1970) tells us that no local hidden-variable theory can reproduce all the predictions of quantum mechanics. Recent experiments in the crucial areas seem to support the predictions of quantum mechanics (d'Espagnat 1979).

It should also be mentioned that Bohm's ideas on hidden variables are, of course, very much more sophisticated. In his discussion of the EPR experiment, for example, he says (Bohm 1952b) that a measurement on one of the particles introduces uncontrollable fluctuations in the wavefunction of the system, which, via so called 'quantum mechanical' forces introduced by Bohm, cause changes in the variables corresponding to the second particle.

Bohm's treatment is in some ways nearer to that appropriate to the Copenhagen interpretation than to that related to less sophisticated hidden-variable theories. It does imagine an observation on one of the particles having a simultaneous effect on the other, which is not necessary on ensemble and simple hidden-variable interpretations. Neither does Bohm attempt to avoid difficulties concerning transmission of forces at infinite velocity between the particles; he merely says that there is no conflict with the theory of special relativity because no 'signal' is carried from one to the other.

Overall, though, and while one recognises these problems for detailed analysis using hidden variables, the situation is clear—when one uses ensemble interpretations there is no difficulty in discussing the basic EPR experiment. Of course, historically, this is to reverse the situation. The object of the EPR paper was to cast doubt on the Copenhagen interpretation, and to encourage its competitors. What is absolutely clear, though, and this is the principal point made in this section, is that any claim to resolve the EPR 'paradox' cannot use the ensemble interpretation. Such a 'resolution' would be completely redundant, as the 'paradox' is simply not paradoxical inside the ensemble interpretation.

## 3. The treatment of Jauch

We now return to the treatment of Jauch (1968), who, as noted above, works almost entirely with density matrices. This is natural enough, of course, because of the rigorous proof of Gleason (1957) that the density matrix is general enough to be used in all quantum-mechanical calculations.

The important division, of course, is between pure states and mixtures. A pure state is represented by an idempotent density matrix, and can also be expressed as a wavefunction. It may correspond to a single system, or an ensemble of systems in the same quantum state. A mixture is represented by a non-idempotent density matrix, and cannot be expressed as a wavefunction. It corresponds to a number of systems in a range of quantum states.

As we have made clear in the previous section, when discussing the EPR experiment we need to consider individual systems, and will thus always be using idempotent density matrices. Alternatively, as stated in I, we may also use wavefunctions, and the two approaches will always give identical results.

The Jauch treatment does not emphasise the question of whether, when using an idempotent density matrix or wavefunction, one is considering a single system or an ensemble. Before the measurement in the EPR experiment the point is irrelevant, but for any discussion of the measurement itself, it is crucial.

Let us imagine that we are working with an ensemble interpretation (or, alternatively, whatever interpretation we are using, that we do not wish to describe the observation process, but merely to describe the state of the ensemble after the observation). We may express this state as a non-idempotent density matrix. We have a mixture because (e.g. Popper 1957), after a measurement, the sub-ensembles must be handled separately. In a general situation of this type, the elements of the density matrix after the measurement will directly correspond to the probabilities of the various results being obtained for each member of the ensemble; this will be particularly obvious if we work in a representation in which the matrix for the observable being measured is diagonal. (If, of course, the wavefunction for the system before the measurement is an eigenfunction of the observable being measured, the state of the system is unchanged by the measurement, so will still be pure after the measurement.)

But the type of calculation described in the last paragraph is emphatically not what is required for our discussion. We must consider a single system. We cannot write down *any* expression for the state of the system after the measurement. (We certainly cannot attempt to write down a mixed state for a single system.) All we can do is to state the probability of getting each of the possible results. Critical discussion of the experiment will then require consideration of the process by which the system changes from its state before the measurement to that after it for *either* of the possible results.

We should make one more general point about Jauch's treatment before discussing it explicitly. He puts great stress on the mathematical treatment of the union and separation of two sub-systems (§11.8). He considers a joint system $S_1+S_2$ composed of two coherent sub-systems $S_1$ and $S_2$. The density matrix of the joint system is $W$, while the reduced density matrices for the sub-systems are $W_1$ and $W_2$. If $W$ is known, $W_1$ can be calculated from

$$W_1 = \mathrm{Tr}_2\, W \tag{1}$$

where $\mathrm{Tr}_2$ denotes a trace over a set of basis states for the Hilbert space of $S_2$. If $A_1$

is an operator representing an observable which only refers to $S_1$, then $A$, the corresponding operator for the combined Hilbert space, is given by

$$A = A_1 \otimes I_2 \tag{2}$$

where $\otimes$ denotes the tensor product (Jauch 1968, §11.7). Then

$$\mathrm{Tr}\, A W = \mathrm{Tr}_1\, A_1 W_1. \tag{3}$$

The particular point expressed by Jauch is that $W$ is not uniquely determined by $W_1$ and $W_2$. We shall see an example shortly. An exception occurs if either $W_1$ or $W_2$ (or both) is pure, when $W$ is uniquely determined by them.

With this fairly lengthy discussion of preliminaries out of the way, we can give an explicit explanation of Jauch's procedure in a rather short space. Before the measurement, the matrix for the system is

$$W = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

with the ordering of states $|++\rangle$, $|+-\rangle$, $|-+\rangle$ and $|--\rangle$. Of course the $|++\rangle$ and $|--\rangle$ states play no part so subsequently we shall omit matrix elements involving them. The reduced density matrices $W_1$ and $W_2$ are each given by

$$W_1 = W_2 = \begin{matrix} \langle +| \\ \langle -| \end{matrix} \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \tag{5}$$

$W$, of course, is pure. (As stated above, it does not matter whether we think of a single system or an ensemble at this stage.) $W_1$ and $W_2$ are non-idempotent, but they are, of course, reduced density matrices.

We now discuss the measurement. At this point it is essential that we decide which interpretation we are using. If we were using the ensemble interpretation, as explained above, we would write for $W$ after the observation

$$W = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \tag{6}$$

(giving only the inner 2×2 block). Again as explained above, $W$ represents a mixture. All this tells us is that, over our ensemble, half of the systems will now be $|+-\rangle$, and half $|-+\rangle$. $W_1$ and $W_2$ are both unchanged from (5), and it is this that Jauch claims as his resolution of the 'paradox'. The system, he claims in effect, has been changed, but neither of the two sub-systems. It seems likely that the argument of CS reduces to the same point.

But, as we emphasised in the last section, if we are to claim to resolve this 'paradox', we must use the Copenhagen interpretation (as defined in §1), and consider an individual observation. The state of the system is *either* $|+-\rangle$ *or* $|-+\rangle$. We have to explain how the system changes from being represented by (4), to being represented by either one of these, or alternatively, how particle 2 (the one that is not directly observed) changes from being represented by (5) to being *one of* $|+\rangle$ and $|-\rangle$.

Of course there is nothing new in this—it is just the EPR 'paradox' as originally presented. The purpose of this paper is to question the claim of Jauch (and that of

CS) to resolve the 'paradox'. Their density-matrix formalism obscures the fact that, implicitly, they seem to be using an ensemble interpretation, inside which *no* resolution is necessary.

## References

Ballentine L E 1970 *Rev. Mod. Phys.* **42** 358
Belinfante F J 1973 *A Survey of Hidden-Variable Theories* (Oxford: Pergamon) p 89
Bell J S 1964 *Physics NY* **1** 195
Bohm D 1951 *Quantum Theory* (New York: Prentice-Hall) ch 22
—— 1952a *Phys. Rev.* **85** 166
—— 1952b *Phys. Rev.* **85** 180
Bohm D and Vigier J P 1954 *Phys. Rev.* **96** 208
Bohr N 1935 *Phys. Rev.* **48** 696
Cantrell C D and Scully M O 1978 *Phys. Rep.* **43** 499
d'Espagnat B 1976 *Conceptual Foundations of Quantum Mechanics* 2nd edn (Reading, Mass.: Benjamin)
—— 1979 *Sci. Am.* **241** 128
Einstein A 1928 *Rapports et Discussions du 5e Conseil Solvay* (Paris: Gauthier-Villars) p 253
—— 1949 in *Albert Einstein: Philosopher–Scientist* ed P A Schilpp (Evanston, Ill: Library of the Living Philosophers) p 663
Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777
Gleason A M 1957 *J. Math. Mech.* **6** 885
Jammer M 1974 *The Philosophy of Quantum Mechanics* (New York: Wiley) p 254
Jauch J M 1968 *Foundations of Quantum Mechanics* (Reading, Mass.: Addison-Wesley) p 189
Popper K S 1957 in *Quantum Theory and Reality* ed M Bunge (London: Butterworth)
Ramachandran G and Murthy M V N 1980 *Nucl. Phys.* A **337** 301
Törnqvist N A 1981 *Found. Phys.* **11** 171
Whitaker M A B and Singh I 1981 *Phys. Lett.* **87A** 9
Wigner E P 1970 *Am. J. Phys.* **38** 1005
—— 1971 *Foundations of Quantum Mechanics* (Enrico Fermi School) ed B d'Espagnat (New York: Academic) p 2